

Chapter 15 - Probability Rules!

1. Homes.

Construct a Venn diagram of the disjoint outcomes.

$$\begin{aligned} \text{a) } P(\text{pool} \cup \text{garage}) &= P(\text{pool}) + P(\text{garage}) - P(\text{pool} \cap \text{garage}) \\ &= 0.64 + 0.21 - 0.17 = 0.68 \end{aligned}$$

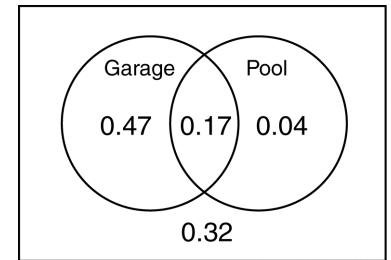
Or, from the Venn: $0.47 + 0.17 + 0.04 = 0.68$

$$\text{b) } P(\text{neither}) = 1 - P(\text{pool} \cup \text{garage}) = 1 - 0.68 = 0.32$$

Or, from the Venn: 0.32 (the region outside the circles)

$$\text{c) } P(\text{pool} \cup \text{no garage}) = P(\text{pool}) - P(\text{pool} \cap \text{garage}) = 0.21 - 0.17 = 0.04$$

Or, from the Venn: 0.04 (the region inside pool circle, yet outside garage circle)



2. Travel.

Construct a Venn diagram of the disjoint outcomes.

$$\begin{aligned} \text{a) } P(\text{Canada} \cap \text{not Mexico}) &= P(\text{Canada}) - P(\text{Canada} \cap \text{Mexico}) = 0.18 - 0.04 = 0.14 \end{aligned}$$

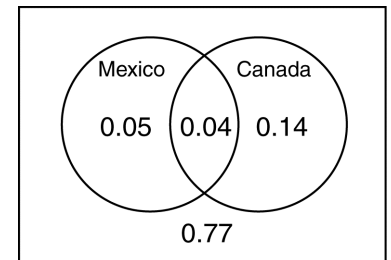
Or, from the Venn: 0.14
(region inside the Canada circle, yet outside the Mexico circle)

$$\begin{aligned} \text{b) } P(\text{Canada} \cup \text{Mexico}) &= P(\text{Canada}) + P(\text{Mexico}) - P(\text{Canada} \cap \text{Mexico}) = 0.18 + 0.09 - 0.04 = 0.23 \end{aligned}$$

Or, from the Venn: $0.05 + 0.04 + 0.14 = 0.23$ (the regions inside the circles)

$$\text{c) } P(\text{neither Canada nor Mexico}) = 1 - P(\text{Canada} \cup \text{Mexico}) = 1 - 0.23 = 0.77$$

Or, from the Venn: 0.77 (the region outside the circles)



3. Amenities.

Construct a Venn diagram of the disjoint outcomes.

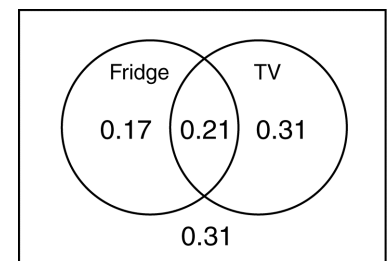
$$\begin{aligned} \text{a) } P(\text{TV} \cap \text{no refrigerator}) &= P(\text{TV}) - P(\text{TV} \cap \text{refrigerator}) \\ &= 0.52 - 0.21 = 0.31 \end{aligned}$$

Or, from the Venn: 0.31
(the region inside the TV circle, yet outside the Fridge circle)

$$\begin{aligned} \text{b) } P(\text{refrigerator or TV, but not both}) &= \\ &= [P(\text{refrigerator}) - P(\text{refrigerator} \cap \text{TV})] + [P(\text{TV}) - P(\text{refrigerator} \cap \text{TV})] \\ &= [0.38 - 0.21] + [0.52 - 0.21] = 0.48 \end{aligned}$$

This problem is much easier to visualize using the Venn diagram. Simply add the probabilities in the two regions for Fridge only and TV only.

$$P(\text{refrigerator or TV, but not both}) = 0.17 + 0.31 = 0.48$$



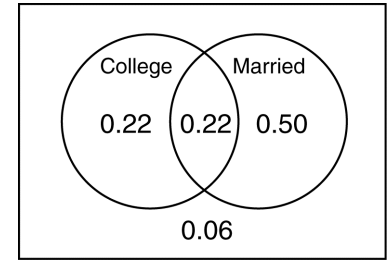
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$$\begin{aligned}
 \text{c) } P(\text{neither TV nor refrigerator}) &= 1 - P(\text{either TV or refrigerator}) \\
 &= 1 - [P(\text{TV}) + P(\text{refrigerator}) - P(\text{TV} \cap \text{refrigerator})] \\
 &= 1 - [0.52 + 0.38 - 0.21] \\
 &= 0.31
 \end{aligned}$$

Or, from the Venn: 0.31 (the region outside the circles)

4. Workers.

Construct a Venn diagram of the disjoint outcomes.



$$\begin{aligned}
 \text{a) } P(\text{neither married nor a college graduate}) &= 1 - P(\text{either married} \cup \text{college graduate}) \\
 &= 1 - [P(\text{married}) + P(\text{college graduate}) - P(\text{both})] \\
 &= 1 - [0.72 + 0.44 - 0.22] \\
 &= 1 - [0.94] \\
 &= 0.06
 \end{aligned}$$

Or, from the Venn: 0.06 (the region outside the circles)

$$\begin{aligned}
 \text{b) } P(\text{married} \cap \text{not a college graduate}) &= P(\text{married}) - P(\text{married} \cap \text{a college graduate}) \\
 &= 0.72 - 0.22 \\
 &= 0.50
 \end{aligned}$$

Or, from the Venn: 0.50 (the region inside the Married circle, yet inside the College circle)

$$\begin{aligned}
 \text{c) } P(\text{married} \cup \text{college graduate}) &= P(\text{married}) + P(\text{college graduate}) - P(\text{both}) \\
 &= 0.72 + 0.44 - 0.22 \\
 &= 0.94
 \end{aligned}$$

Or, from the Venn diagram: $0.22 + 0.22 + 0.50 = 0.94$ (the regions inside the circles)

5. Global survey.

$$\text{a) } P(\text{USA}) = \frac{1557}{7690} \approx 0.2025$$

$$\text{b) } P(\text{some high school} \cup \text{primary or less}) = \frac{4195}{7690} + \frac{1161}{7690} \approx 0.6965$$

c)

$$\begin{aligned}
 P(\text{France} \cup \text{post-graduate}) &= P(\text{France}) + P(\text{post-graduate}) - P(\text{both}) \\
 &= \frac{1539}{7690} + \frac{379}{7690} - \frac{69}{7690} \approx 0.2404
 \end{aligned}$$

$$\text{d) } P(\text{France} \cap \text{primary school or less}) = \frac{309}{7690} \approx 0.0402$$

6. Birth order.

$$\text{a) } P(\text{Human Ecology}) = \frac{43}{223} \approx 0.193$$

$$\text{b) } P(\text{first - born}) = \frac{113}{223} \approx 0.507$$

c) $P(\text{first-born} \cap \text{Human Ecology}) = \frac{15}{223} \approx 0.067$

d)

$$\begin{aligned} &P(\text{first-born} \cup \text{Human Ecology}) \\ &= P(\text{first-born}) + P(\text{Human Ecology}) - P(\text{first-born} \cap \text{Human Ecology}) \\ &= \frac{113}{223} + \frac{43}{223} - \frac{15}{223} \approx 0.632 \end{aligned}$$

7. Cards.

a) $P(\text{heart} \mid \text{red}) = \frac{P(\text{heart} \cap \text{red})}{P(\text{red})} = \frac{13/52}{26/52} = \frac{1}{2}$

A more intuitive approach is to think about only the red cards. Half of them are hearts.

b) $P(\text{red} \mid \text{heart}) = \frac{P(\text{red} \cap \text{heart})}{P(\text{heart})} = \frac{13/52}{13/52} = 1$

Think about only the hearts. They are all red!

c) $P(\text{ace} \mid \text{red}) = \frac{P(\text{ace} \cap \text{red})}{P(\text{red})} = \frac{2/52}{26/52} = \frac{2}{26} \approx 0.077$

Consider only the red cards. Of those 26 cards, 2 of them are aces.

d) $P(\text{queen} \mid \text{face}) = \frac{P(\text{queen} \cap \text{face})}{P(\text{face})} = \frac{4/52}{12/52} \approx 0.333$

There are 12 faces cards: 4 jacks, 4 queens, and 4 kings. Four of the 12 face cards are queens.

8. Pets.

Organize the counts in a two-way table.

a) $P(\text{male} \mid \text{cat}) = \frac{P(\text{male} \cap \text{cat})}{P(\text{cat})} = \frac{6/42}{18/42} = \frac{6}{18} \approx 0.333$

	Cats	Dogs	Total
Male	6	8	14
Female	12	16	28
Total	18	24	42

Consider only the Cats column. There are 6 male cats, out of a total of 18 cats.

b) $P(\text{cat} \mid \text{female}) = \frac{P(\text{cat} \cap \text{female})}{P(\text{female})} = \frac{12/42}{28/42} = \frac{12}{28} \approx 0.429$

We are interested in the Female row. Of the 28 female animals, 12 are cats.

c) $P(\text{female} \mid \text{dog}) = \frac{P(\text{female} \cap \text{dog})}{P(\text{dog})} = \frac{16/42}{24/42} = \frac{16}{24} \approx 0.667$

Look at only the Dogs column. There are 24 dogs, and 16 of them are female.

9. Health.

Construct a two-way table of the conditional probabilities, including the marginal probabilities.

		Blood Pressure		
		High	OK	Total
Cholesterol	High	0.11	0.21	0.32
	OK	0.16	0.52	0.68
	Total	0.27	0.73	1.00

a) $P(\text{both conditions}) = 0.11$

b) $P(\text{high blood pressure}) = 0.11 + 0.16 = 0.27$

c) $P(\text{high chol.} \mid \text{high BP}) = \frac{P(\text{high chol.} \cap \text{high BP})}{P(\text{high BP})} = \frac{0.11}{0.27} \approx 0.407$

Consider only the High Blood Pressure column. Within this column, the probability of having high cholesterol is 0.11 out of a total of 0.27.

d) $P(\text{high BP} \mid \text{high chol.}) = \frac{P(\text{high BP} \cap \text{high chol.})}{P(\text{high chol.})} = \frac{0.11}{0.32} \approx 0.344$

This time, consider only the high cholesterol row. Within this row, the probability of having high blood pressure is 0.11, out of a total of 0.32.

10. Death penalty.

a) Construct a two-way table of the conditional probabilities, including the marginal probabilities.

		Death Penalty		
		Favor	Oppose	Total
Party	Republican	0.26	0.04	0.30
	Democrat	0.12	0.24	0.36
	Other	0.24	0.10	0.34
	Total	0.62	0.38	1.00

i) $P(\text{favor the death penalty}) = 0.26 + 0.12 + 0.24 = 0.62$

ii) $P(\text{favor death penalty} \mid \text{Republican}) = \frac{P(\text{favor death penalty} \cap \text{Rep.})}{P(\text{Republican})} = \frac{0.26}{0.30} \approx 0.867$

Consider only the Republican row. The probability of favoring the death penalty is 0.26 out of a total of 0.30 for that row.

iii) $P(\text{Democrat} \mid \text{favor death penalty}) = \frac{P(\text{Democrat} \cap \text{favor death penalty})}{P(\text{favor death penalty})} = \frac{0.12}{0.62} \approx 0.194$

Consider only the Favor column. The probability of being a Democrat is 0.12 out of a total of 0.62 for that column.

$$\begin{aligned} \text{b) } P(\text{Republican} \cup \text{or favor death penalty}) &= P(\text{Republican}) + P(\text{favor death pen.}) - P(\text{both}) \\ &= 0.30 + 0.62 - 0.26 \\ &= 0.66 \end{aligned}$$

The overall probabilities of being a Republican and favoring the death penalty are from the marginal distribution of probability (the totals). The candidate can expect 66% of the votes, provided her estimates are correct.

11. Global survey, take 2.

$$\text{a) } P(\text{USA and postgraduate work}) = \frac{84}{7690} \approx 0.011$$

$$\text{b) } P(\text{USA} \mid \text{post-graduate}) = \frac{84}{379} \approx 0.222$$

$$\text{c) } P(\text{post-graduate} \mid \text{USA}) = \frac{84}{1557} \approx 0.054$$

$$\text{d) } P(\text{primary} \mid \text{China}) = \frac{506}{1502} \approx 0.337$$

$$\text{e) } P(\text{China} \mid \text{primary}) = \frac{506}{1161} \approx 0.436$$

12. Birth order, take 2.

$$\text{a) } P(\text{Arts and Science and second child}) = \frac{23}{223} \approx 0.103$$

$$\text{b) } P(\text{second child} \mid \text{Arts and Science}) = \frac{23}{57} \approx 0.404$$

$$\text{c) } P(\text{Arts and Science} \mid \text{second child}) = \frac{23}{110} \approx 0.209$$

$$\text{d) } P(\text{Agriculture} \mid \text{first - born}) = \frac{52}{113} \approx 0.460$$

$$\text{e) } P(\text{first - born} \mid \text{Agriculture}) = \frac{52}{93} \approx 0.559$$

13. Sick kids.

Having a fever and having a sore throat are not independent events, so:

$$P(\text{fever} \cap \text{and sore throat}) = P(\text{Fever}) P(\text{Sore Throat} \mid \text{Fever}) = (0.70)(0.30) = 0.21$$

The probability that a kid with a fever has a sore throat is 0.21.

14. Sick cars.

Needing repairs and paying more than \$400 for the repairs are not independent events. (What happens to the probability of paying more than \$400, if you don't need repairs?!)

$$\begin{aligned} &P(\text{needing repairs} \cap \text{paying more than } \$400) \\ &= P(\text{needing repairs}) P(\text{paying more than } \$400 \mid \text{repairs are needed}) \\ &= (0.20)(0.40) = 0.08 \end{aligned}$$

15. Cards.

a) $P(\text{first heart drawn is on the third card}) = P(\text{no heart})P(\text{no heart})P(\text{heart})$

$$= \left(\frac{39}{52}\right)\left(\frac{38}{51}\right)\left(\frac{13}{50}\right) \approx 0.145$$

b) $P(\text{all three cards drawn are red}) = P(\text{red})P(\text{red})P(\text{red})$

$$= \left(\frac{26}{52}\right)\left(\frac{25}{51}\right)\left(\frac{24}{50}\right) \approx 0.118$$

c) $P(\text{none of the cards are spades}) = P(\text{no spade})P(\text{no spade})P(\text{no spade})$

$$= \left(\frac{39}{52}\right)\left(\frac{38}{51}\right)\left(\frac{37}{50}\right) \approx 0.414$$

d) $P(\text{at least one of the cards is an ace}) = 1 - P(\text{none of the cards are aces})$

$$\begin{aligned} &= 1 - [P(\text{no ace})P(\text{no ace})P(\text{no ace})] \\ &= 1 - \left(\frac{48}{52}\right)\left(\frac{47}{51}\right)\left(\frac{46}{50}\right) \approx 0.217 \end{aligned}$$

16. Another hand.

a) $P(\text{none of the cards are aces}) = P(\text{no ace})P(\text{no ace})P(\text{no ace})$

$$= \left(\frac{48}{52}\right)\left(\frac{47}{51}\right)\left(\frac{46}{50}\right) \approx 0.783$$

b) $P(\text{all of the cards are hearts}) = P(\text{heart})P(\text{heart})P(\text{heart})$

$$= \left(\frac{13}{52}\right)\left(\frac{12}{51}\right)\left(\frac{11}{50}\right) \approx 0.013$$

c) $P(\text{the third card is the first red}) = P(\text{no red})P(\text{no red})P(\text{red})$

$$= \left(\frac{26}{52}\right)\left(\frac{25}{51}\right)\left(\frac{26}{50}\right) \approx 0.127$$

$$\begin{aligned}
 \text{d)} \quad P(\text{at least one of the cards is a diamond}) &= 1 - P(\text{none of the cards are diamonds}) \\
 &= 1 - [P(\text{no diam.})P(\text{no diam.})P(\text{no diam.})] \\
 &= 1 - \left(\frac{39}{52}\right)\left(\frac{38}{51}\right)\left(\frac{37}{50}\right) \approx 0.586
 \end{aligned}$$

17. Batteries.

Since batteries are not being replaced, use conditional probabilities throughout.

$$\begin{aligned}
 \text{a)} \quad P(\text{the first two batteries are good}) &= P(\text{good})P(\text{good}) \\
 &= \left(\frac{7}{12}\right)\left(\frac{6}{11}\right) \approx 0.318
 \end{aligned}$$

$$\begin{aligned}
 \text{b)} \quad P(\text{at least one of the first three batteries works}) &= 1 - P(\text{none of the first three batt. work}) \\
 &= 1 - [P(\text{no good})P(\text{no good})P(\text{no good})] \\
 &= 1 - \left(\frac{5}{12}\right)\left(\frac{4}{11}\right)\left(\frac{3}{10}\right) \approx 0.955
 \end{aligned}$$

$$\begin{aligned}
 \text{c)} \quad P(\text{the first four batteries are good}) &= P(\text{good})P(\text{good})P(\text{good})P(\text{good}) \\
 &= \left(\frac{7}{12}\right)\left(\frac{6}{11}\right)\left(\frac{5}{10}\right)\left(\frac{4}{9}\right) \approx 0.071
 \end{aligned}$$

$$\begin{aligned}
 \text{d)} \quad P(\text{pick five to find one good}) &= P(\text{no good})P(\text{no good})P(\text{no good})P(\text{no good})P(\text{good}) \\
 &= \left(\frac{5}{12}\right)\left(\frac{4}{11}\right)\left(\frac{3}{10}\right)\left(\frac{2}{9}\right)\left(\frac{7}{8}\right) \approx 0.009
 \end{aligned}$$

18. Shirts.

You need two shirts so don't replace them. Use conditional probabilities throughout.

$$\begin{aligned}
 \text{a)} \quad P(\text{the first two are not mediums}) &= P(\text{not medium})P(\text{not medium}) \\
 &= \left(\frac{16}{20}\right)\left(\frac{15}{19}\right) \approx 0.632
 \end{aligned}$$

$$\begin{aligned}
 \text{b)} \quad P(\text{the first medium is the third shirt}) &= P(\text{no medium})P(\text{no medium})P(\text{medium}) \\
 &= \left(\frac{16}{20}\right)\left(\frac{15}{19}\right)\left(\frac{4}{18}\right) \approx 0.140
 \end{aligned}$$

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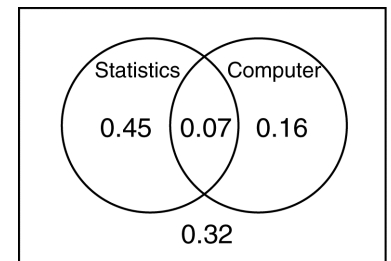
c) $P(\text{the first four shirts are extra - large}) = P(\text{XL}) P(\text{XL}) P(\text{XL}) P(\text{XL})$
 $= \left(\frac{6}{20}\right)\left(\frac{5}{19}\right)\left(\frac{4}{18}\right)\left(\frac{3}{17}\right) \approx 0.003$

d) $P(\text{at least one of four is a med.}) = 1 - P(\text{none of the first four shirts are mediums})$
 $= 1 - [P(\text{no med.}) P(\text{no med.}) P(\text{no med.}) P(\text{no med.})]$
 $= 1 - \left(\frac{16}{20}\right)\left(\frac{15}{19}\right)\left(\frac{14}{18}\right)\left(\frac{13}{17}\right) \approx 0.624$

19. Eligibility.

Construct a Venn diagram of the disjoint outcomes.

a) $P(\text{eligibility}) = P(\text{statistics}) + P(\text{computer science}) - P(\text{both})$
 $= 0.52 + 0.23 - 0.07$
 $= 0.68$



68% of students are eligible for BioResearch, so $100 - 68 = 32\%$ are ineligible.

From the Venn, the region outside the circles represents those students who have taken neither course, and are therefore ineligible for BioResearch.

b) $P(\text{computer science} \mid \text{statistics}) = \frac{P(\text{computer science} \cap \text{statistics})}{P(\text{statistics})} = \frac{0.07}{0.52} \approx 0.135$

From the Venn, consider only the region inside the Statistics circle. The probability of having taken computer science is 0.07 out of a total of 0.52 (the entire Statistics circle).

- c) Taking the two courses are not disjoint events, since they have outcomes in common. In fact, 7% of juniors have taken both courses.
- d) Taking the two courses are not independent events. The overall probability that a junior has taken a computer science is 0.23. The probability that a junior has taken a computer course given that he or she has taken a statistics course is 0.135. If taking the two courses were independent events, these probabilities would be the same.

20. Benefits.

Construct a Venn diagram of the disjoint outcomes.

a) $P(\text{neither benefit}) = 1 - P(\text{either retirement} \cup \text{health})$
 $= 1 - [P(\text{retirement}) + P(\text{health}) - P(\text{both})]$
 $= 1 - [0.56 + 0.68 - 0.49]$
 $= 0.25$



b)

$$P(\text{health insurance} \mid \text{retirement}) = \frac{P(\text{health insurance} \cap \text{retirement})}{P(\text{retirement})} = \frac{0.49}{0.56} = 0.875$$

From the Venn, consider only the region inside the Retirement circle. The probability that a worker has health insurance is 0.49 out of a total of 0.56 (the entire Retirement circle).

- c) Having health insurance and a retirement plan are not independent events. 68% of all workers have health insurance, while 87.5% of workers with retirement plans also have health insurance. If having health insurance and a retirement plan were independent events, these percentages would be the same.
- d) Having these two benefits are not disjoint events, since they have outcomes in common. 49% of workers have both health insurance and a retirement plan.

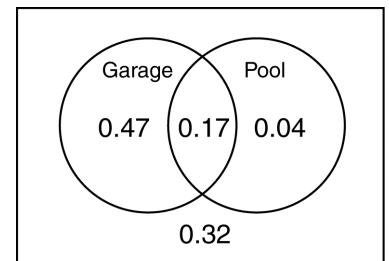
21. For sale.

Construct a Venn diagram of the disjoint outcomes.

a)

$$P(\text{pool} \mid \text{garage}) = \frac{P(\text{pool} \cap \text{garage})}{P(\text{garage})} = \frac{0.17}{0.64} \approx 0.266$$

From the Venn, consider only the region inside the Garage circle. The probability that the house has a pool is 0.17 out of a total of 0.64 (the entire Garage circle).



- b) Having a garage and a pool are not independent events. 26.6% of homes with garages have pools. Overall, 21% of homes have pools. If having a garage and a pool were independent events, these would be the same.
- c) No, having a garage and a pool are not disjoint events. 17% of homes have both.

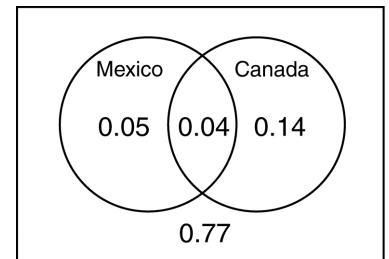
22. On the road again.

Construct a Venn diagram of the disjoint outcomes.

a)

$$P(\text{Canada} \mid \text{Mexico}) = \frac{P(\text{Canada} \cap \text{Mexico})}{P(\text{Mexico})} = \frac{0.04}{0.09} \approx 0.444$$

From the Venn, consider only the region inside the Mexico circle. The probability that an American has traveled to Canada is 0.04 out of a total of 0.09 (the entire Mexico circle).



- b) No, travel to Mexico and Canada are not disjoint events. 4% of Americans have been to both countries.
- c) No, travel to Mexico and Canada are not independent events. 18% of U.S. residents have been to Canada. 44.4% of the U.S. residents who have been to Mexico have also been to Canada. If travel to the two countries were independent, the percentages would be the same.

23. Cards.

Yes, getting an ace is independent of the suit when drawing one card from a well shuffled deck. The overall probability of getting an ace is $4/52$, or $1/13$, since there are 4 aces in the deck. If you consider just one suit, there is only 1 ace out of 13 cards, so the probability of getting an ace given that the card is a diamond, for instance, is $1/13$. Since the probabilities are the same, getting an ace is independent of the suit.

24. Pets again.

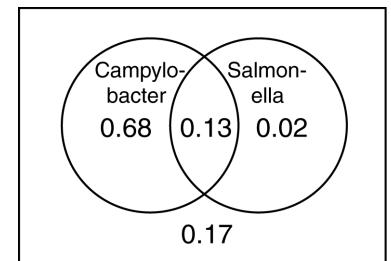
Consider the two-way table from Exercise 8.

	Cats	Dogs	Total
Male	6	8	14
Female	12	16	28
Total	18	24	42

Yes, species and gender are independent events. 8 of 24, or $1/3$ of the dogs are male, and 6 of 18, or $1/3$ of the cats are male. Since these are the same, species and gender are independent events.

25. Unsafe food.

- a) Using the Venn diagram, the probability that a tested chicken was not contaminated with either kind of bacteria is 17%.
- b) Contamination with campylobacter and contamination with salmonella are not disjoint events, since 13% of chicken are contaminated with both.



- c) No, contamination with campylobacter and contamination with salmonella are not independent events. The probability that a tested chicken is contaminated with campylobacter is 0.81. The probability that chicken contaminated with salmonella is also contaminated with campylobacter is $0.13/0.15 = 0.87$. If chicken is contaminated with salmonella, it is more likely to be contaminated with campylobacter than chicken in general.

26. Birth order, finis.

- a) Yes, since the events share no outcomes. Students can enroll in only one college.
- b) No, since knowing that one event is true drastically changes the probability of the other. The probability of a student being in the Agriculture college is nearly 42%. The probability of a student being in the Human Ecology college, given that he or she is in the Agriculture college is 0.
- c) No, since they share outcomes. 15 students were first-born, Human Ecology students.
- d) No, since knowing that one event is true drastically changes the probability of the other. Over 19% of all students enrolled in Human Ecology, but only 13% of first-borns did.

27. Men's health, again.

Consider the two-way table from Exercise 9.

High blood pressure and high cholesterol are not independent events. 28.8% of men with OK blood pressure have high cholesterol, while 40.7% of men with high blood pressure have high cholesterol. If having high blood pressure and high cholesterol were independent, these percentages would be the same.

		Blood Pressure		Total
		High	OK	
Cholesterol	High	0.11	0.21	0.32
	OK	0.16	0.52	0.68
Total		0.27	0.73	1.00

28. Politics.

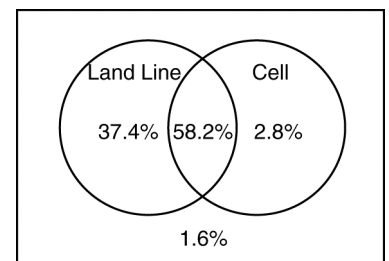
Consider the two-way table from Exercise 10.

Party affiliation and position on the death penalty are not independent events. 86.7% of Republicans favor the death penalty, but only 33.3% of Democrats favor it. If the events were independent, then these percentages would be the same.

		Death Penalty		Total
		Favor	Oppose	
Party	Republican	0.26	0.04	0.30
	Democrat	0.12	0.24	0.36
	Other	0.24	0.10	0.34
	Total	0.62	0.38	1.00

29. Phone service.

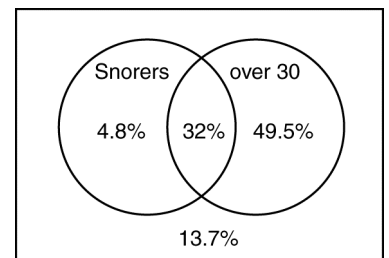
- a) Since 2.8% of U.S. adults have only a cell phone, and 1.6% have no phone at all, polling organizations can reach $100 - 2.8 - 1.6 = 96.5\%$ of U.S. adults.
- b) Using the Venn diagram, about 96.5% of U.S. adults have a land line. The probability of a U.S. adults having a land line given that they have a cell phone is $58.2 / (58.2 + 2.8)$ or about 95.4%. It appears that having a cell phone and having a land line are independent, since the probabilities are roughly the same.



30. Snoring.

Organize the percentages in a Venn diagram.

- a) 13.7% of the respondents were under 30 and did not snore.
- b) According to this survey, snoring is not independent of age. 36.8% of the 995 adults snored, but $32 / (32 + 49.5) = 39.3\%$ of those over 30 snored.



31. Montana.

According to the poll, party affiliation is not independent of sex. Overall, $(36+48)/202 = 41.6\%$ of the respondents were Democrats. Of the men, only $36/105 = 34.3\%$ were Democrats.

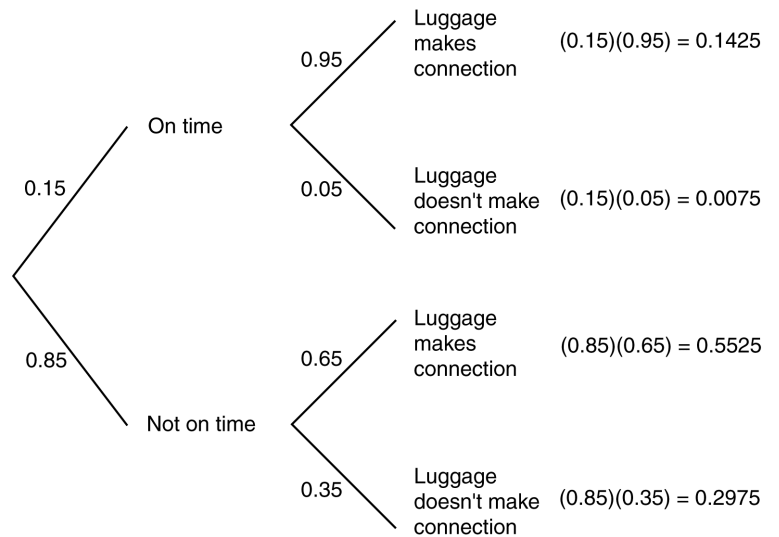
32. Cars.

According to the survey, country of origin of the car is not independent of type of driver. $(33+12)/359 = 12.5\%$ of the cars were of European origin, but about $33/195 = 16.9\%$ of the students drive European cars.

33. Luggage.

Organize using a tree diagram.

- a) No, the flight leaving on time and the luggage making the connection are not independent events. The probability that the luggage makes the connection is dependent on whether or not the flight is on time. The probability is 0.95 if the flight is on time, and only 0.65 if it is not on time.

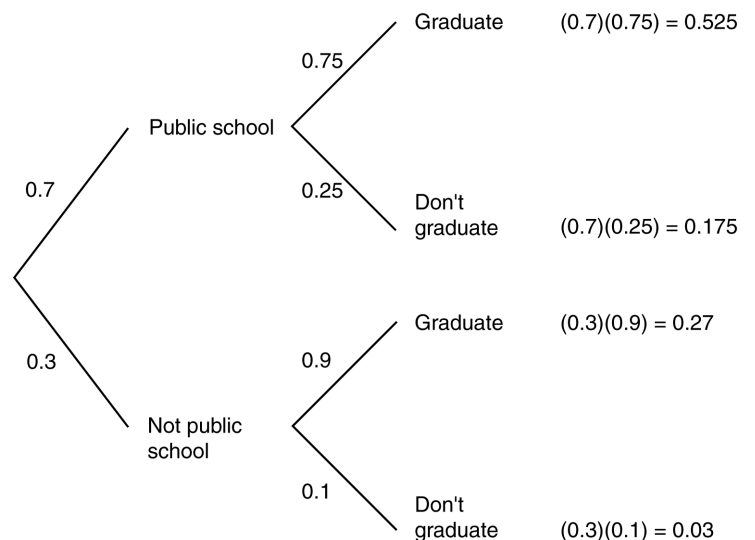


- b)

$$\begin{aligned}
 P(\text{Luggage}) &= P(\text{On time} \cap \text{Luggage}) + P(\text{Not on time} \cap \text{Luggage}) \\
 &= (0.15)(0.95) + (0.85)(0.65) \\
 &= 0.695
 \end{aligned}$$

34. Graduation.

- a) Yes, there is evidence to suggest that a freshman's chances to graduate depend upon what kind of high school the student attended. The graduation rate for public school students is 75%, while the graduation rate for others is 90%. If the high school attended was independent of college graduation, these percentages would be the same.



b)

$$\begin{aligned}
 P(\text{Graduate}) &= P(\text{Public} \cap \text{Graduate}) + P(\text{Not public} \cap \text{Graduate}) \\
 &= (0.7)(0.75) + (0.3)(0.9) \\
 &= 0.795
 \end{aligned}$$

Overall, 79.5% of freshmen are expected to eventually graduate.

35. Late luggage.

Refer to the tree diagram constructed for Exercise 33.

$$P(\text{Not on time} \mid \text{No Lug.}) = \frac{P(\text{Not on time} \cap \text{No Lug.})}{P(\text{No Lug.})} = \frac{(0.85)(0.35)}{(0.15)(0.05) + (0.85)(0.35)} \approx 0.975$$

If you pick Leah up at the Denver airport and her luggage is not there, the probability that her first flight was delayed is 0.975.

36. Graduation, part II.

Refer to the tree diagram constructed for Exercise 34.

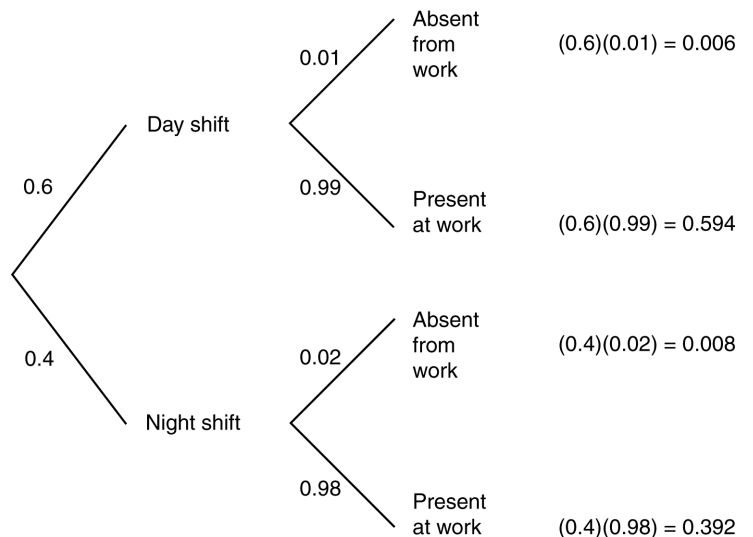
$$P(\text{Public} \mid \text{Graduate}) = \frac{P(\text{Public} \cap \text{Graduate})}{P(\text{Graduate})} = \frac{(0.7)(0.75)}{(0.7)(0.75) + (0.3)(0.9)} \approx 0.660$$

Overall, 66.0% of the graduates of the private college went to public high schools.

37. Absenteeism.

Organize the information in a tree diagram.

- a) No, absenteeism is not independent of shift worked. The rate of absenteeism for the night shift is 2%, while the rate for the day shift is only 1%. If the two were independent, the percentages would be the same.



b)

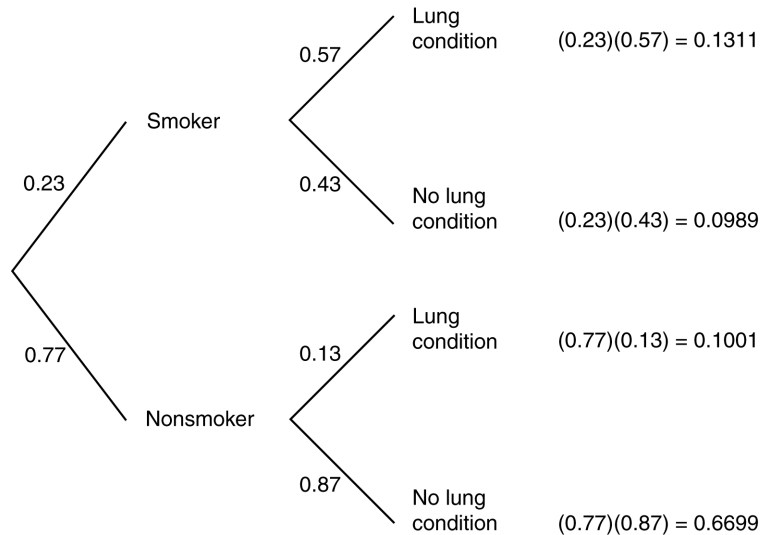
$$P(\text{Absent}) = P(\text{Day} \cap \text{Absent}) + P(\text{Night} \cap \text{Absent}) = (0.6)(0.01) + (0.4)(0.02) = 0.014$$

The overall rate of absenteeism at this company is 1.4%.

38. Lungs and smoke.

Organize the information into a tree diagram.

- a) The lung condition and smoking are not independent, since rates of the lung condition are different for smokers and nonsmokers. 57% of smokers have the lung condition by age 60, while only 13% of nonsmokers have the condition by age 60.



- b)

$$\begin{aligned}
 &P(\text{Lung condition}) \\
 &= P(\text{Smoker} \cap \text{Lung Condition}) + P(\text{Nonsmoker} \cap \text{Lung Condition}) \\
 &= (0.23)(0.57) + (0.77)(0.13) \\
 &\approx 0.231
 \end{aligned}$$

The probability that a randomly selected 60-year-old has the lung condition is about 0.231.

39. Absenteeism, part II.

Refer to the tree diagram constructed for Exercise 37.

$$P(\text{Night} \mid \text{Absent}) = \frac{P(\text{Night} \cap \text{Absent})}{P(\text{Absent})} = \frac{(0.4)(0.02)}{(0.6)(0.01) + (0.4)(0.02)} \approx 0.571$$

Approximately 57.1% of the company's absenteeism occurs on the night shift.

40. Lungs and Smoke, again.

Refer to the tree diagram constructed for Exercise 38.

$$P(\text{Smoker} \mid \text{Lung cond.}) = \frac{P(\text{Smoker} \cap \text{Lung cond.})}{P(\text{Lung cond.})} = \frac{(0.23)(0.57)}{(0.23)(0.57) + (0.77)(0.13)} \approx 0.567$$

The probability that someone who has the lung condition by age 60 is a smoker is approximately 56.7%.

41. Drunks.

Organize the information into a tree diagram.



a) $P(\text{Detain} \mid \text{Not Drinking}) = 0.2$

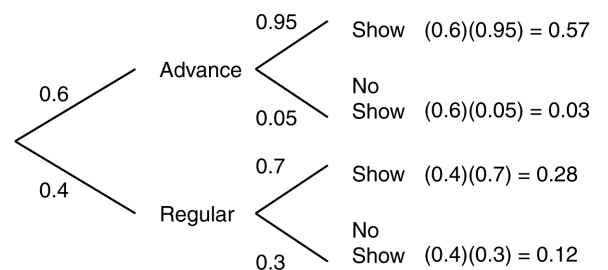
b)
$$\begin{aligned} P(\text{Detain}) &= P(\text{Drinking} \cap \text{Det.}) \\ &\quad + P(\text{Not Drinking} \cap \text{Det.}) \\ &= (0.12)(0.8) + (0.88)(0.2) \\ &= 0.272 \end{aligned}$$

c)
$$\begin{aligned} P(\text{Drunk} \mid \text{Det.}) &= \frac{P(\text{Drunk} \cap \text{Det.})}{P(\text{Detain})} \\ &= \frac{(0.12)(0.8)}{(0.12)(0.8) + (0.88)(0.2)} \\ &\approx 0.353 \end{aligned}$$

d)
$$\begin{aligned} P(\text{Drunk} \mid \text{Release}) &= \frac{P(\text{Drunk} \cap \text{Release})}{P(\text{Release})} \\ &= \frac{(0.12)(0.2)}{(0.12)(0.2) + (0.88)(0.8)} \\ &\approx 0.033 \end{aligned}$$

42. No-shows.

Organize the information into a tree diagram.



a)
$$\begin{aligned} P(\text{No Show}) &= P(\text{Advance} \cap \text{No Show}) \\ &\quad + P(\text{Regular} \cap \text{No Show}) \\ &= (0.60)(0.05) + (0.40)(0.30) \\ &= 0.03 + 0.12 = 0.15 \end{aligned}$$

b)
$$P(\text{Advance} \mid \text{No Show}) = \frac{P(\text{Advance} \cap \text{No Show})}{P(\text{No Show})} = \frac{0.03}{0.15} = 0.20$$

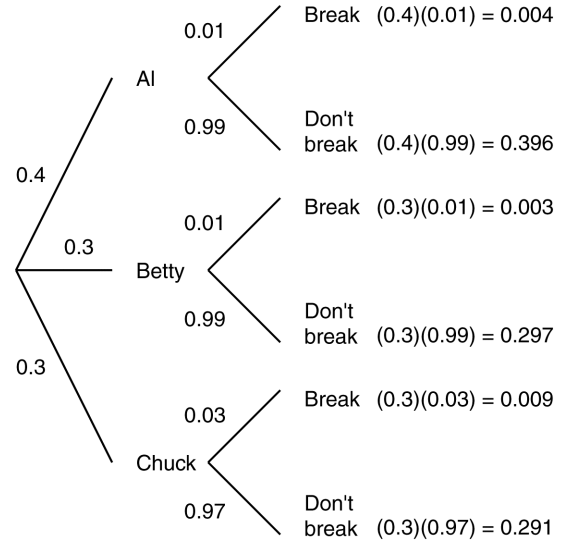
c) No, being a no show is not independent of the type of ticket a passenger holds. While 30% of regular fare passengers are no shows, only 5% of advanced sale fare passengers are no shows.

43. Dishwashers.

Organize the information in a tree diagram.

$$\begin{aligned}
 &P(\text{Chuck} \mid \text{Break}) \\
 &= \frac{P(\text{Chuck} \cap \text{Break})}{P(\text{Break})} \\
 &= \frac{(0.3)(0.03)}{(0.4)(0.01) + (0.3)(0.01) + (0.3)(0.03)} \\
 &\approx 0.563
 \end{aligned}$$

If you hear a dish break, the probability that Chuck is on the job is approximately 0.563.

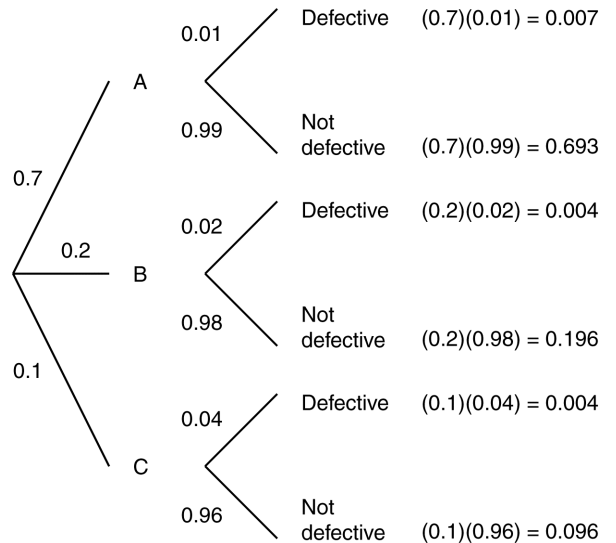


44. Parts.

Organize the information in a tree diagram.

$$\begin{aligned}
 &P(\text{Supplier A} \mid \text{Defective}) \\
 &= \frac{P(\text{Supplier A} \cap \text{Defective})}{P(\text{Defective})} \\
 &= \frac{(0.7)(0.01)}{(0.7)(0.01) + (0.2)(0.02) + (0.1)(0.04)} \\
 &\approx 0.467
 \end{aligned}$$

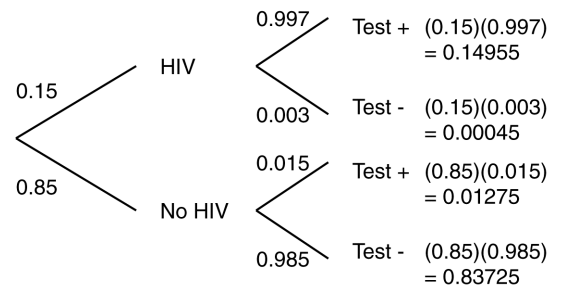
The probability that a defective component came from supplier A is approximately 0.467.



45. HIV Testing.

Organize the information in a tree diagram.

$$\begin{aligned}
 P(\text{No HIV} \mid \text{Test -}) &= \frac{P(\text{No HIV} \cap \text{Test -})}{P(\text{Test -})} \\
 &= \frac{0.83725}{0.00045 + 0.83725} \\
 &\approx 0.999
 \end{aligned}$$



The probability that a patient testing negative is truly free of HIV is about 99.9%.

46. Polygraphs.

Organize the information in a tree diagram.

$$\begin{aligned}
 &P(\text{Trustworthy} \mid \text{"Lie" on poly.}) \\
 &= \frac{P(\text{Trustworthy} \cap \text{"Lie" on poly.})}{P(\text{"Lie" on poly.})} \\
 &= \frac{(0.95)(0.15)}{(0.95)(0.15) + (0.05)(0.65)} \\
 &\approx 0.814
 \end{aligned}$$

The probability that a job applicant rejected under suspicion of dishonesty is actually trustworthy is about 0.814.

